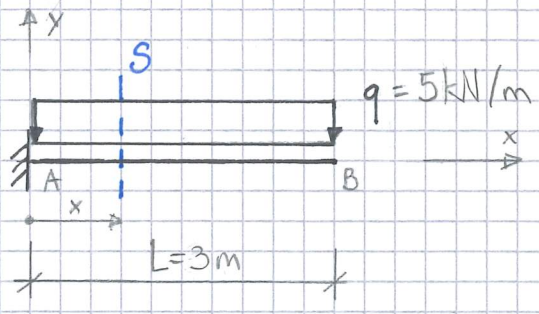
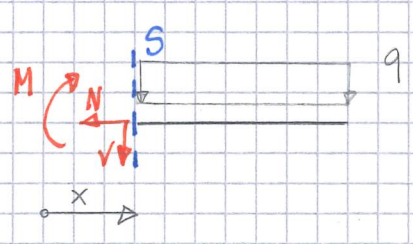
i)




Poutre console encastrée en A et libre en B
Une seule zone d'étude → 1 seule coupe S
En travaillant sur la partie de droite il n'est pas nécessaire de déterminer les 3 réactions d'appui : $A_x$, $A_y$ et $M_A$ en A.




$\Sigma F_x = 0 : N(x) = 0$

$\Sigma F_y = 0 : -V(x) - q.(L-x) = 0 \Rightarrow V(x) = -q(L-x)$

en $x = 0 : V = -q.L = -5.3 = -15$ kN

en $x = L : V = 0$

$\Sigma M = 0 : -M(x) - q.(L-x)\frac{(L-x)}{2} = 0 \Rightarrow M(x) = -q\frac{(L-x)^2}{2}$

en $x = 0 : M = -q\frac{L^2}{2} = -5.\frac{3^2}{2} = -22{,}5$ kNm

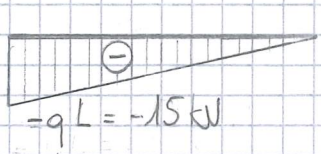
en $x = L : M = 0$

en $x = \frac{L}{2} : M = -q\frac{L^2}{8} = -5.\frac{3^2}{8} = -5{,}63$ kNm

N

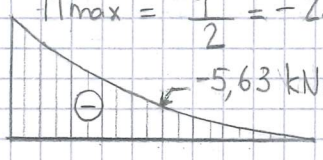



V

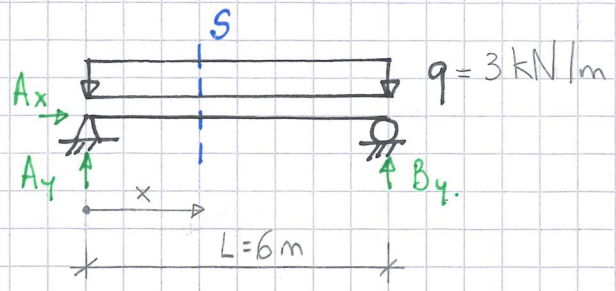



$M_{max} = \frac{-qL^2}{2} = -22{,}5$ kNm

M



j).



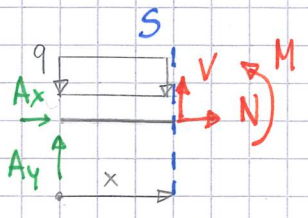

Appui fixe en A $\Rightarrow A_x, A_y$ et appui mobile en B $\Rightarrow B_y$

La poutre est symétrique et uniformément chargée $\Rightarrow A_y = B_y = \frac{qL}{2} = 9\,kN$

$A_x = 0$

Diagramme des efforts intérieurs

Une seule zone d'étude est à considérer $\rightarrow$ 1 seule coupe.




$\Sigma F_x = 0 : A_x + N(x) = 0 \Rightarrow N(x) = 0$

$\Sigma F_y = 0 : A_y - q.x + V(x) = 0 \Rightarrow V(x) = -A_y + q.x$

en $x = 0 : V = -A_y = -\frac{qL}{2} = -9\,kN$

$x = \frac{L}{2} : V = -A_y + q\frac{L}{2} = 0$

$x = L : V = -A_y + qL = \frac{qL}{2} = 9\,kN$

$\Sigma M = 0 : -A_y.x + q.x.\frac{x}{2} + M(x) = 0 \Rightarrow M(x) = A_y.x - q\frac{x^2}{2}$

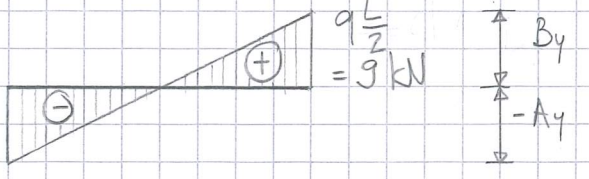
en $x = 0 : M = 0$

$x = \frac{L}{2} : M = A_y.\frac{L}{2} - q\frac{\left(\frac{L}{2}\right)^2}{2} = q\frac{L^2}{8} = 13{,}5\,kNm$

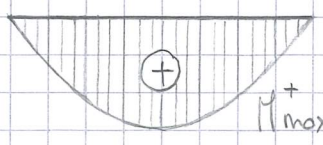
$x = L : M = 0$

N

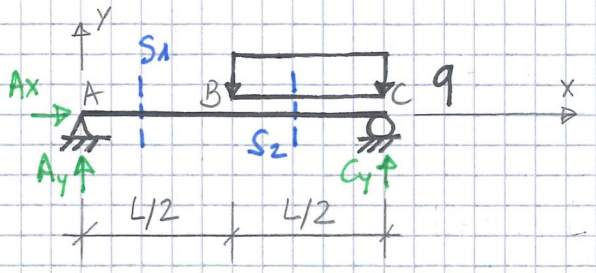

V




M



k)




$q = 4\ kN/m$ et $L = 6\ m$

## Réactions d'appui

$\Sigma F_x = 0 : A_x = 0$

$\Sigma F_y = 0 : A_y - q\frac{L}{2} + C_y = 0 \Rightarrow A_y = q\frac{L}{2} - C_y$

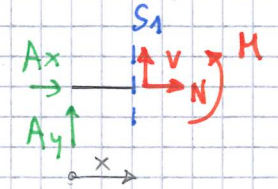
$\Sigma M_A = 0 : -q\frac{L}{2}\cdot\frac{3L}{4} + C_y \cdot L = 0 \Rightarrow C_y = \frac{3}{4} q \frac{L}{2} = \frac{3}{4}\cdot 4 \cdot \frac{6}{2} = 9\ kN$

$A_y = q\frac{L}{2} - \frac{3}{4} q\frac{L}{2} = \frac{1}{4} q\frac{L}{2} = \frac{1}{4}\cdot 4 \cdot \frac{6}{2} = 3\ kN$

## Diagrammes des efforts intérieurs

Deux zones à considérer : AB et BC $\Rightarrow$ 2 Coupes ($S_1, S_2$)

Partie AB :




$\Sigma F_x = 0 : A_x + N(x) = 0 \Rightarrow N(x) = 0$

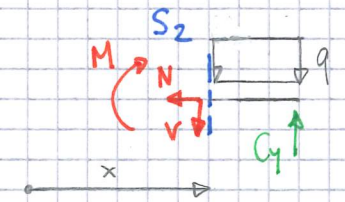
$\Sigma F_y = 0 : A_y + V(x) = 0 \Rightarrow V(x) = -A_y = -3\ kN$

$\Sigma M = 0 : -A_y \cdot x + M(x) = 0 \Rightarrow M(x) = A_y \cdot x = \frac{1}{4} q \frac{L}{2} \cdot x$

en $x = 0 : M = 0$

en $x = \frac{L}{2} : M = \frac{1}{4} q \frac{L}{2} \cdot \frac{L}{2} = \frac{1}{16} q L^2 = 9\ kNm$

Partie BC




$\Sigma F_x = 0 : -N(x) = 0$

$\Sigma F_y = 0 : -V(x) - q(L-x) + C_y = 0 \Rightarrow V(x) = C_y - q(L-x)$

en $x = \frac{L}{2} : V = C_y - q\frac{L}{2} = -A_y = -3\ kN$

en $x = L : V = C_y = 9\ kN$

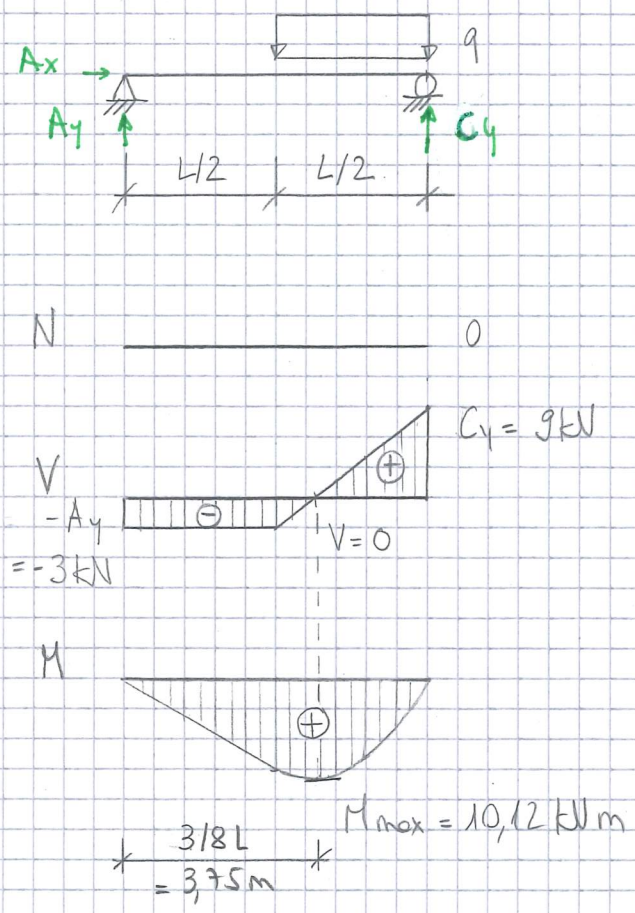
$\Sigma M = 0 : -M(x) - q(L-x)\frac{(L-x)}{2} + C_y \cdot (L-x) = 0$

$\Rightarrow M(x) = C_y(L-x) - q\frac{(L-x)^2}{2}$

en $x = \frac{L}{2} : M = C_y \cdot \frac{L}{2} - q\cdot\frac{(L/2)^2}{2} = \frac{1}{16} q L^2 = \frac{1}{16}\cdot 4 \cdot 6^2 = 9\ kNm$

en $x = L : M = C_y \cdot 0 - q\cdot\frac{0^2}{2} = 0$

suite k).

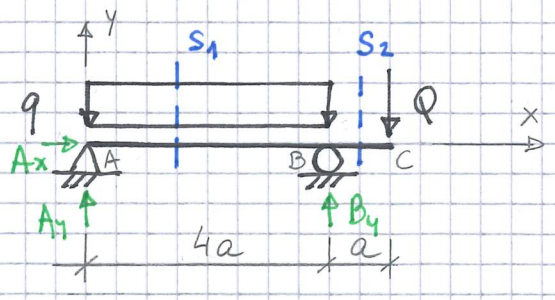



Point particulier : D. où $V=0$

$$V(x) = C_y - q(L-x) = 0 \Rightarrow x = L - \frac{C_y}{q} = L - \frac{\frac{3}{4}q\frac{L}{2}}{q} = \frac{5}{8}L = 3{,}75\,m$$

$$\Rightarrow M = C_y\left(L-\frac{5}{8}L\right) - q\frac{\left(L-\frac{5}{8}L\right)^2}{2} = \frac{9}{128}qL^2 = \frac{9}{128}4.6^2 = 10{,}12\,kNm$$

l)




$q = 4\,kN/m$ ; $Q = 6\,kN$

$a = 2m$

## Réactions d'appui

$\Sigma F_x = 0 : A_x = 0$

$\Sigma F_y = 0 : A_y - q.4a + B_y - Q = 0 \Rightarrow A_y = q.4a + Q - B_y$

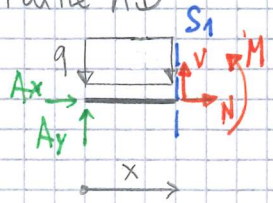
$\Sigma M_A = 0 : -q.4a.2a + B_y.4a - Q.5a = 0 \Rightarrow B_y = \frac{1}{4}(q8a + 5Q) = q2a + \frac{5}{4}Q = 23,5\,kN$

$A_y = q2a - \frac{Q}{4} = 14,5\,kN$

## Diagrammes des efforts intérieurs

Deux zones à étudier : AB et BC $\Rightarrow$ 2 coupes ($S_1$, $S_2$)

### Partie AB




$\Sigma F_x = 0 : A_x + N(x) = 0 \Rightarrow N(x) = 0$

$\Sigma F_y = 0 : A_y - q.x + V(x) = 0 \Rightarrow V(x) = -A_y + qx$

en $x = 0$ : $V = -A_y = -14,5\,kN$

en $x = 4a$ : $V = -A_y + q.4a = 17,5\,kN$

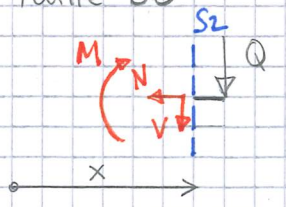
$\Sigma M = 0 : -A_y.x + q.x\frac{x}{2} + M(x) = 0 \Rightarrow M(x) = A_y.x - q\frac{x^2}{2}$

en $x = 0$ : $M = 0$

en $x = 4a$ : $M = A_y.4a - q\frac{(4a)^2}{2} = -Qa = -12\,kNm$

(en $x = 2a$ : $M = A_y.2a - q\frac{(2a)^2}{2} = 2qa^2 - \frac{Qa}{2} = 26\,kNm$)

### Partie BC




$\Sigma F_x = 0 : N(x) = 0$

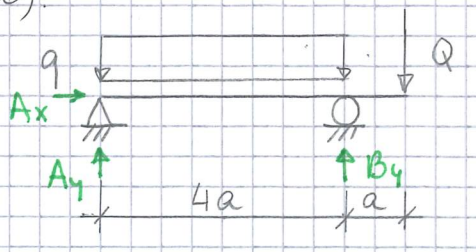
$\Sigma F_y = 0 : -V(x) - Q = 0 \Rightarrow V(x) = -Q = -6\,kN$

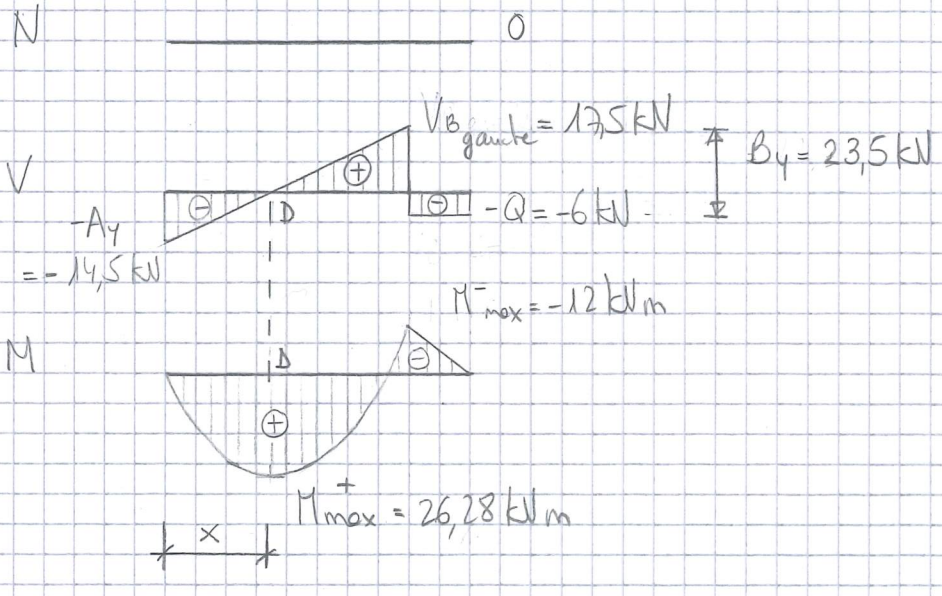
$\Sigma M = 0 : -M(x) - Q(5a - x) = 0 \Rightarrow M(x) = -Q(5a - x)$

en $x = 4a$ : $M = -Q.a = -12\,kNm$

en $x = 5a$ : $M = 0$

suite l).




Point particulier : D où $V = 0$

$$V(x) = -A_y + qx = 0 \Rightarrow x = \frac{A_y}{q} = \frac{14,5}{4} = 3,625\,m$$

$$\Rightarrow M = A_y \cdot x - q\frac{x^2}{2} = 14,5 \cdot 3,625 - 4\frac{3,625^2}{2} = 26,28\,kN.m$$