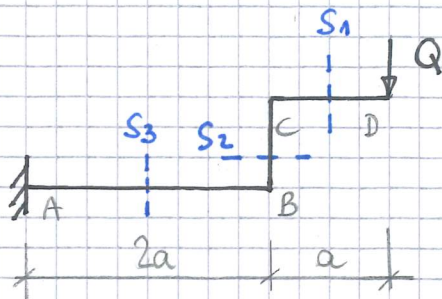


e)

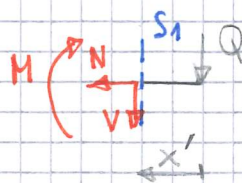


$$Q = 3 \text{ kN}$$

$$a = 1 \text{ m}$$

Il s'agit d'une poutre courbée en B et en C encastrée en A et libre en D.
Trois zones d'études sont à considérer : AB, BC et CD $\rightarrow$ 3 coupes (S_3, S_2, S_1)
En travaillant sur les parties de droite il n'est pas nécessaire de déterminer les réactions d'appui A_x, A_y et M_A en A.

Partie CD



$$\sum F_x = 0 : -N(x') = 0$$

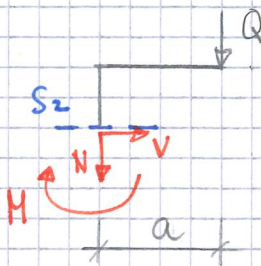
$$\sum F_y = 0 : -V(x') - Q = 0 \Rightarrow V(x') = -Q = -3 \text{ kN}$$

$$\sum M = 0 : -M(x') - Q \cdot x' = 0 \Rightarrow M(x') = -Q \cdot x'$$

$$\text{en D } (x' = 0) : M = 0$$

$$\text{en C } (x' = a) : M = -Q \cdot a = -3 \cdot 1 = -3 \text{ kNm}$$

Partie CB

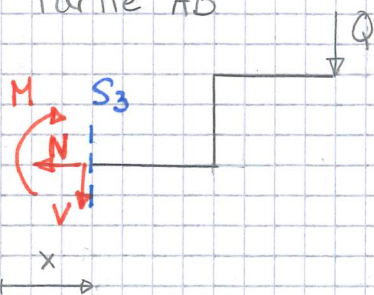


$$\sum F_x = 0 : V(x) = 0$$

$$\sum F_y = 0 : -N(x) - Q = 0 \Rightarrow N(x) = -Q = -3 \text{ kN}$$

$$\sum M = 0 : -M(x) - Q \cdot a = 0 \Rightarrow M(x) = -Q \cdot a = -3 \cdot 1 = -3 \text{ kNm}$$

Partie AB



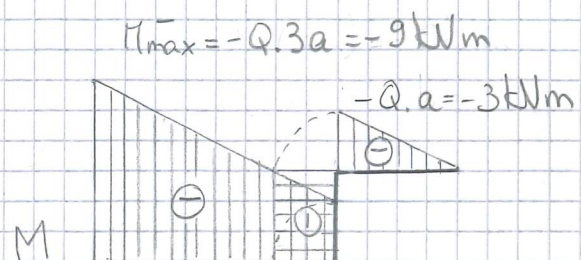
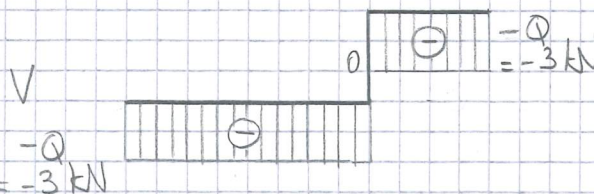
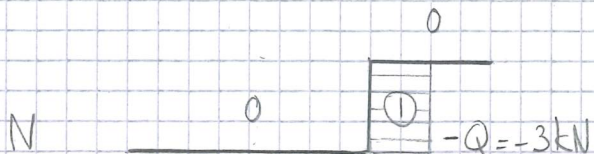
$$\sum F_x = 0 : -N(x) = 0$$

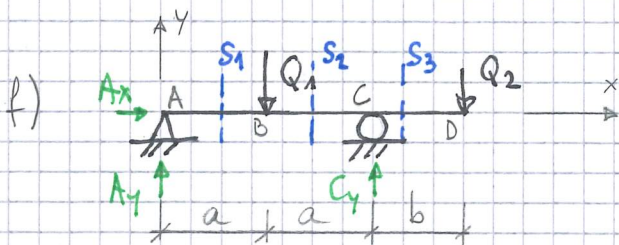
$$\sum F_y = 0 : -V(x) - Q = 0 \Rightarrow V(x) = -Q = -3 \text{ kN}$$

$$\sum M = 0 : -M(x) - Q \cdot (3a - x) = 0 \Rightarrow M(x) = -Q \cdot (3a - x)$$

$$\text{en } x = 0 : M = -Q \cdot 3a = -3 \cdot 3 = -9 \text{ kNm}$$

$$\text{en } x = a : M = -Q \cdot a = -3 \cdot 1 = -3 \text{ kNm}$$





$$Q_1 = 15 \text{ kN}, Q_2 = 10 \text{ kN}$$

$$a = 3,5 \text{ m}, b = 3 \text{ m}$$

Réactions d'appui

$$\sum F_x = 0 : A_x = 0$$

$$\sum F_y = 0 : A_y + Q_1 + C_y - Q_2 = 0 \Rightarrow C_y = Q_1 + Q_2 - A_y$$

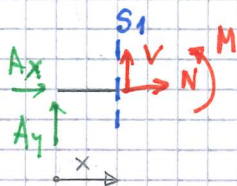
$$\sum M_C = 0 : -A_y \cdot 2a + Q_1 \cdot a - Q_2 \cdot b = 0 \Rightarrow A_y = \frac{Q_1 \cdot a - Q_2 \cdot b}{2a} = \frac{15 \cdot 3,5 - 10 \cdot 3}{2 \cdot 3,5} = 3,2 \text{ kN}$$

$$C_y = 15 + 10 - 3,2 = 21,8 \text{ kN}$$

Diagrammes des efforts intérieurs

Trois zones à considérer : AB, BC et CD $\rightarrow$ 3 coupes (S_1, S_2, S_3)

Partie AB



$$\sum F_x = 0 : A_x + N(x) = 0 \Rightarrow N(x) = 0$$

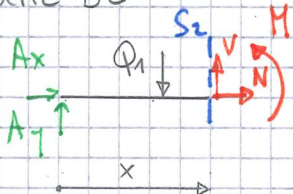
$$\sum F_y = 0 : A_y + V(x) = 0 \Rightarrow V(x) = -A_y = -3,2 \text{ kN}$$

$$\sum M = 0 : -A_y \cdot x + M(x) = 0 \Rightarrow M(x) = A_y \cdot x$$

$$\text{en } x=0(A) : M=0$$

$$\text{en } x=a(B) : M = A_y \cdot a = 11,2 \text{ kNm}$$

Partie BC



$$\sum F_x = 0 : N(x) = 0$$

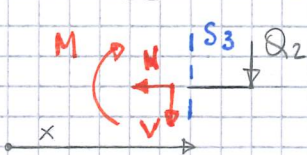
$$\sum F_y = 0 : A_y - Q_1 + V(x) = 0 \Rightarrow V(x) = Q_1 - A_y = \frac{Q_1 \cdot a + Q_2 \cdot b}{2a} = 11,8 \text{ kN}$$

$$\sum M = 0 : -A_y \cdot x + Q_1 \cdot (x-a) + M(x) = 0 \Rightarrow M(x) = A_y \cdot x - Q_1 \cdot (x-a)$$

$$\text{en } x=a(B) : M = A_y \cdot a = 11,2 \text{ kNm}$$

$$\text{en } x=2a(C) : M = A_y \cdot 2a - Q_1 \cdot a = -30 \text{ kNm}$$

Partie CD



$$\sum F_x = 0 : N(x) = 0$$

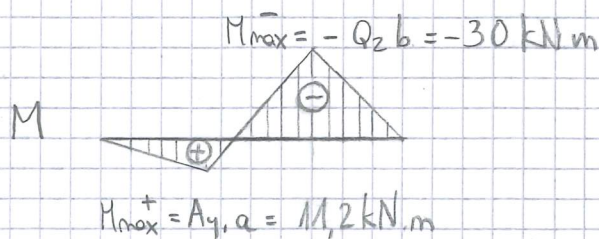
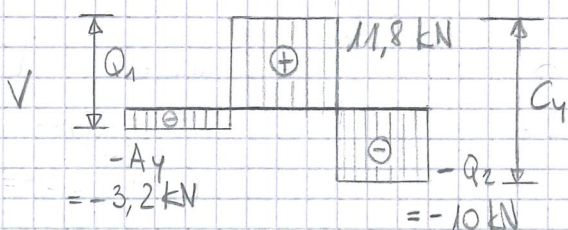
$$\sum F_y = 0 : -V(x) - Q_2 = 0 \Rightarrow V(x) = -Q_2 = -10 \text{ kN}$$

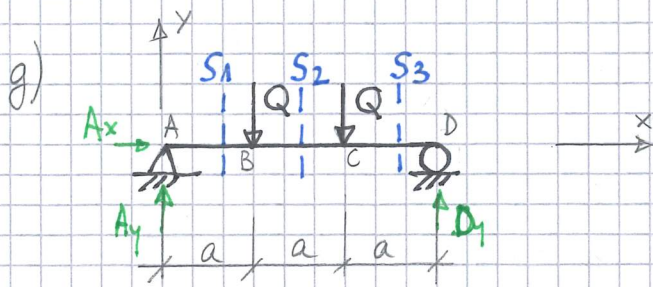
$$\sum M = 0 : -M(x) - Q_2 \cdot (2a+b-x) = 0 \Rightarrow M(x) = -Q_2 \cdot (2a+b-x)$$

$$\text{en } x=2a(C) : M = -Q_2 \cdot b = -30 \text{ kNm}$$

$$\text{en } x=2a+b(D) : M=0$$

N





$$Q = 10 \text{ kN}, a = 2 \text{ m}$$

Réactions d'appui

$$\sum F_x = 0: A_x = 0$$

$$\sum F_y = 0: A_y - Q - Q + D_y = 0 \Rightarrow A_y = 2Q - D_y$$

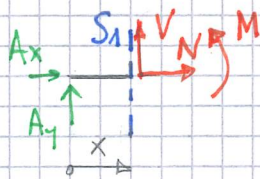
$$\sum M_A = 0: -Q \cdot a - Q \cdot 2a + D_y \cdot 3a = 0 \Rightarrow D_y = Q = 10 \text{ kN}$$

$$\Rightarrow A_y = 2Q - Q = Q = 10 \text{ kN}$$

Diagrammes des efforts intérieurs

Trois zones sont à considérer: les parties AB, BC et CD $\rightarrow$ 3 coupes (S_1, S_2, S_3)

Partie AB



$$\sum F_x = 0: A_x + N(x) = 0 \Rightarrow N(x) = -A_x = 0$$

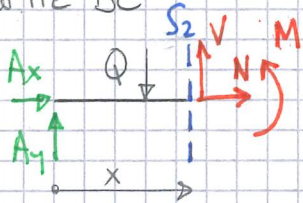
$$\sum F_y = 0: A_y + V(x) = 0 \Rightarrow V(x) = -A_y = -Q = -10 \text{ kN}$$

$$\sum M = 0: -A_y \cdot x + M(x) = 0 \Rightarrow M(x) = A_y \cdot x = Q \cdot x$$

$$\text{en } x = 0 \text{ (A): } M = 0$$

$$\text{en } x = a \text{ (B): } M = Q \cdot a = 10 \cdot 2 = 20 \text{ kNm}$$

Partie BC



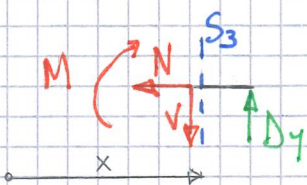
$$\sum F_x = 0: A_x + N(x) = 0 \Rightarrow N(x) = -A_x = 0$$

$$\sum F_y = 0: A_y - Q + V(x) = 0 \Rightarrow V(x) = -A_y + Q = -Q + Q = 0$$

$$\sum M = 0: -A_y \cdot x + Q \cdot (x - a) + M(x) = 0 \Rightarrow M(x) = Q \cdot a$$

$$\text{entre } x = a \text{ (B) et } x = 2a \text{ (C): } M = Q \cdot a = 20 \text{ kNm}$$

Partie CD



$$\sum F_x = 0: N(x) = 0$$

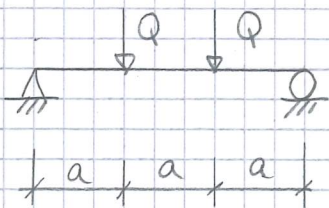
$$\sum F_y = 0: -V(x) + D_y = 0 \Rightarrow V(x) = D_y = Q$$

$$\sum M = 0: -M(x) + D_y(3a - x) = 0 \Rightarrow M(x) = Q \cdot (3a - x)$$

$$\text{en } x = 2a \text{ (C): } M = Q \cdot a = 20 \text{ kNm}$$

$$x = 3a \text{ (D): } M = 0$$

suite g)



N _____ 0



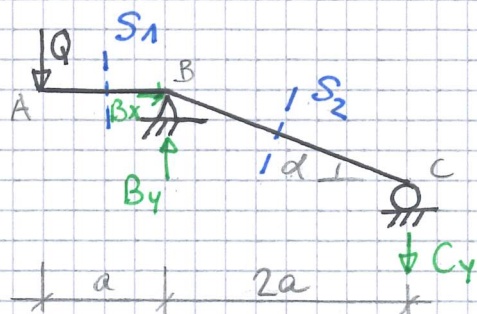
$\uparrow D_y$
 $\downarrow -A_y$

M



$$M_{\max}^+ = Qa = 20 \text{ kN.m}$$

h)



Réactions d'appui

$$\sum F_x = 0 : B_x = 0$$

$$\sum F_y = 0 : -Q + B_y - C_y = 0 \Rightarrow B_y = Q + C_y$$

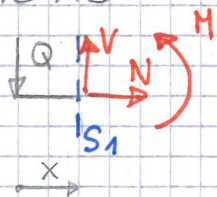
$$\sum M_B = 0 : -Q \cdot a - C_y \cdot 2a = 0 \Rightarrow C_y = \frac{Q}{2}$$

$$\Rightarrow B_y = Q + \frac{Q}{2} = \frac{3Q}{2}$$

Diagramme des efforts intérieurs

Deux zones sont à étudier : les parties AB et BC $\rightarrow$ 2 coupes (S_1, S_2)

Partie AB



$$\sum F_x = 0 : N(x) = 0$$

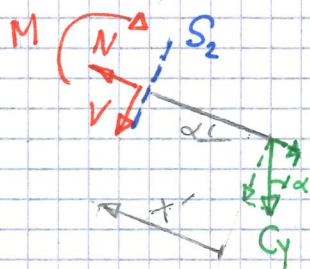
$$\sum F_y = 0 : -Q + V(x) = 0 \Rightarrow V(x) = Q$$

$$\sum M = 0 : Q \cdot x + M(x) = 0 \Rightarrow M(x) = -Qx$$

$$\text{en } x=0 : M=0$$

$$\text{en } x=a : M = -Qa$$

Partie BC



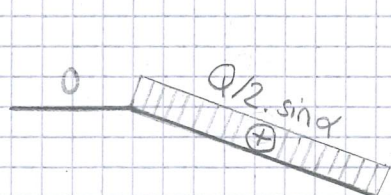
$$\sum F_x = 0 : -N(x') + C_y \sin \alpha = 0 \Rightarrow N(x') = C_y \sin \alpha = \frac{Q}{2} \sin \alpha$$

$$\sum F_y = 0 : -V(x') - C_y \cos \alpha = 0 \Rightarrow V(x') = -C_y \cos \alpha = -\frac{Q}{2} \cos \alpha$$

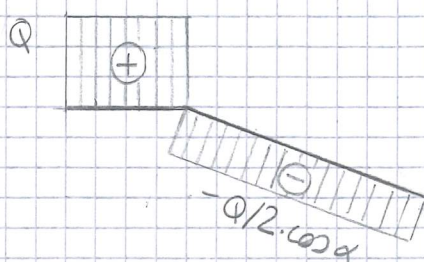
$$\sum M = 0 : -M(x') - C_y \cos \alpha \cdot x' = 0 \Rightarrow M(x') = -C_y \cos \alpha \cdot x' = -\frac{Q}{2} \cos \alpha \cdot x'$$

$$\text{en } x'=0 : M=0$$

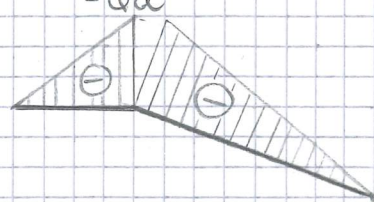
$$\text{en } x'=\frac{2a}{\cos \alpha} : M = -\frac{Q}{2} \cos \alpha \cdot \frac{2a}{\cos \alpha} = -Qa$$



N



V



M