h e p i a
Haute école du paysage, d'ingénierie
et d'architecture de Genève

Rue de la Prairie 4
CH-1202 Genève

Tél. +41 (0)22 546 24 00
Fax +41 (0)22 546 24 10

hepia@hesge.ch
www.hesge.ch/hepia

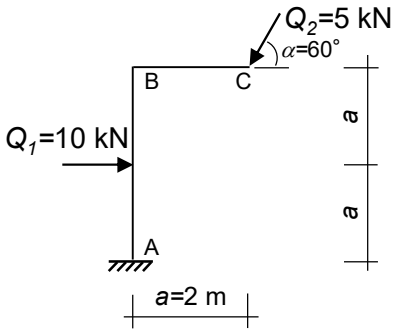
Unité de cours TB_231 – Epreuve 1 – 04.12.2020

9h00-11h15 – Documents papier autorisés

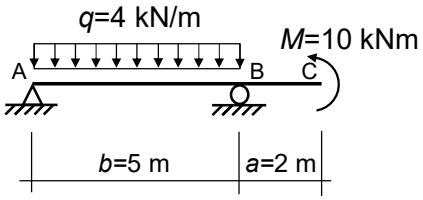
Exercice 1

Calculer les réactions d'appuis des poutres suivantes (3 pts par poutre).

a)



b)

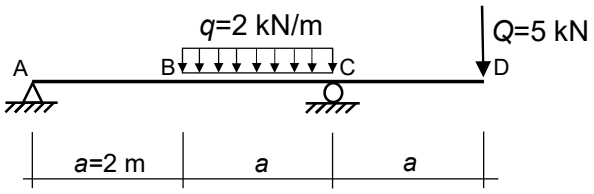


Exercice 2

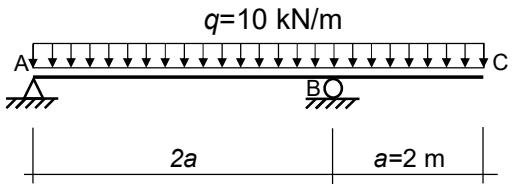
Déterminer pour les poutres suivantes :

- Les réactions d'appuis (2 pts par poutre).
- Les expressions de l'effort tranchant et du moment de flexion le long de la poutre (1 pt par zone par poutre).
- Les diagrammes de l'effort tranchant et du moment de flexion en précisant les valeurs en A, B, C, D (1 pt par diagramme par poutre).

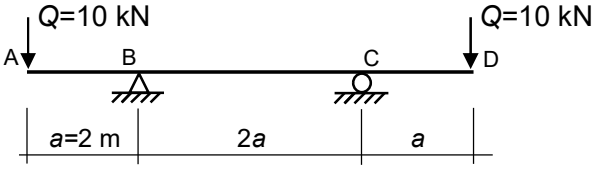
a)



b)



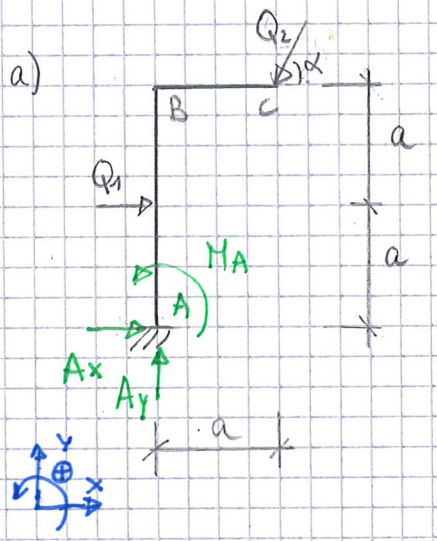
c)



TB 231 - Correction Epreuve 1 04.12.2020 Durée 2h15

Exercice 1.
Calculer les réactions d'appui

a)



$Q_1 = 10$ kN, $Q_2 = 5$ kN
$a = 2$ m
$\alpha = 60°$

Encastrement en A ⇒ 3 inconnues A_x, A_y, M_A

$\Sigma F_x = 0$: $A_x + Q_1 - Q_2 \cos\alpha = 0$

$\Rightarrow A_x = -Q_1 + Q_2 \cos\alpha$

$= -10 + 5 \cdot \cos 60° = -7{,}5$ kN

$\Sigma F_y = 0$: $A_y - Q_2 \sin\alpha = 0$

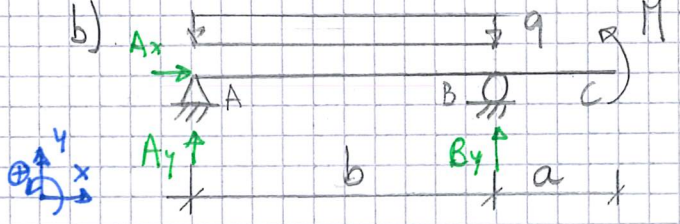
$\Rightarrow A_y = Q_2 \sin\alpha = 5 \cdot \sin 60° = 4{,}33$ kN

$\Sigma M_A = 0$: $M_A - Q_1 \cdot a + Q_2 \cos\alpha \cdot 2a - Q_2 \sin\alpha \cdot a = 0$

$\Rightarrow M_A = Q_1 a - Q_2 \cos\alpha \cdot 2a + Q_2 \sin\alpha \cdot a$

$= 10 \cdot 2 - 5 \cdot \cos 60 \cdot 2 \cdot 2 + 5 \cdot \sin 60 \cdot 2 = 18{,}66$ kNm

b)



$q = 4$ kN/m ; $M = 10$ kNm
$a = 2$ m, $b = 5$ m

Appui fixe en A ⇒ 2 inconnues A_x, A_y
Appui mobile en B ⇒ 1 inconnue B_y

$\Sigma F_x = 0$: $A_x = 0$

$\Sigma F_y = 0$: $A_y - q \cdot b + B_y = 0 \Rightarrow A_y = qb - B_y$

$\Sigma M_A = 0$: $-q \cdot b \cdot \frac{b}{2} + B_y \cdot b + M = 0 \Rightarrow B_y = \frac{1}{b}\left(q\frac{b^2}{2} - M\right)$

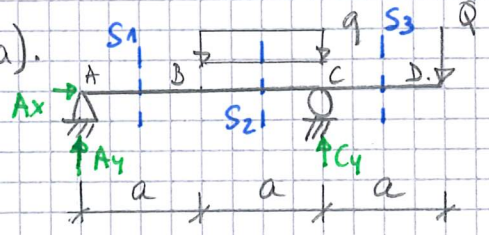
$= \frac{1}{5}\left(4 \cdot \frac{5^2}{2} - 10\right) = 8$ kN.

$\Rightarrow A_y = qb - B_y = 4 \cdot 5 - 8 = 12$ kN.

Exercice 2.

Calculer les réactions d'appui, les expressions de V et M et les diagrammes V et M.

a).



$q = 2\ kN/m$; $Q = 5\ kN$

$a = 2m$

Réactions d'appui

$\Sigma F_x = 0$: $A_x = 0$

$\Sigma F_y = 0$: $A_y - q.a + C_y - Q = 0 \Rightarrow A_y = q.a - C_y + Q$.

$\Sigma M_A = 0$: $-q.a.\left(a + \frac{a}{2}\right) + C_y.2a - Q.3a = 0 \Rightarrow C_y = \frac{1}{2a}\left(q.\frac{3}{2}a^2 + Q3a\right)$

$= \frac{1}{2.2}\left(2.\frac{3}{2}.2^2 + 5.3.2\right) = 10,5\ kN$

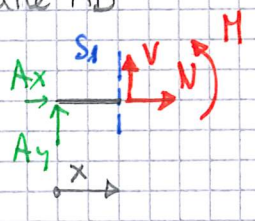
$\Rightarrow A_y = 2.2 - 10,5 + 5 = -1,5\ kN$

A_y agit vers le bas

Diagramme des efforts intérieurs

Trois zones à considérer : AB, BC et CD → 3 coupes (S_1, S_2, S_3)

Partie AB



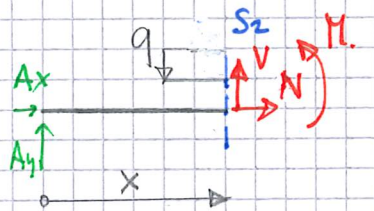
$\Sigma F_y = 0$: $A_y + V(x) = 0 \Rightarrow V(x) = -A_y = -(-1,5) = 1,5\ kN$

$\Sigma M = 0$: $-A_y.x + M(x) = 0 \Rightarrow M(x) = A_y.x$

en $x = 0$: $M = 0$

en $x = a$: $M = A_y.a = -1,5.2 = -3\ kNm$

Partie BC



$\Sigma F_y = 0$: $A_y - q(x-a) + V(x) = 0 \Rightarrow V(x) = -A_y + q(x-a)$

en $x = a$: $V = -A_y + q.0 = -A_y = -(-1,5) = 1,5\ kN$

en $x = 2a$: $V = -A_y + qa = -(-1,5) + 2.2 = 5,5\ kN$

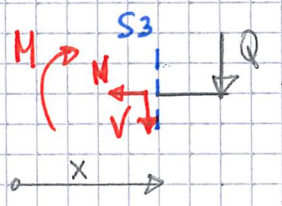
$\Sigma M = 0$: $-A_y.x + q.(x-a)\frac{(x-a)}{2} + M(x) = 0 \Rightarrow M(x) = A_y.x - q\frac{(x-a)^2}{2}$

en $x = a$: $M = A_y.a - q.0 = -1,5.2 = -3\ kNm$

en $x = 2a$: $M = A_y.2a - q.\frac{a^2}{2} = -1,5.2.2 - 2.\frac{2^2}{2}$

$= -10\ kNm$

Partie CD.

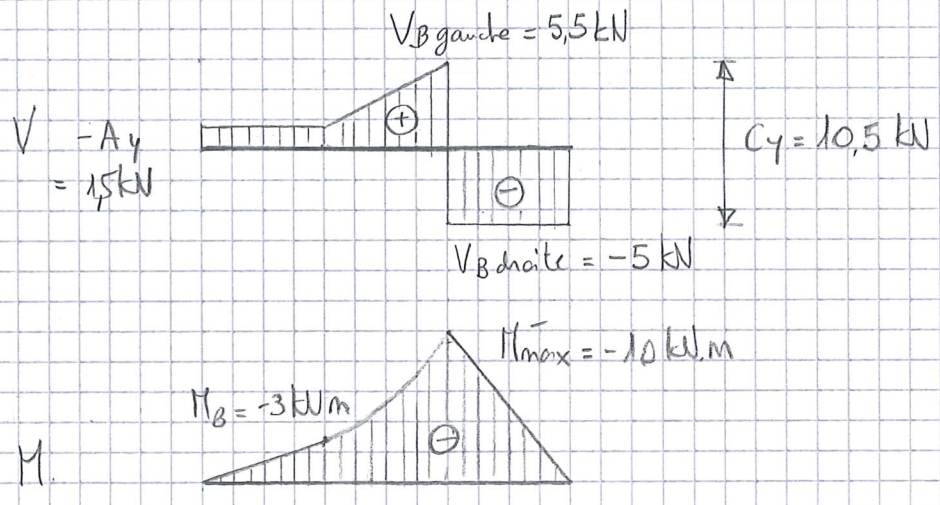


$\Sigma F_y = 0 : -V(x) - Q = 0 \Rightarrow V(x) = -Q = -5\,kN$

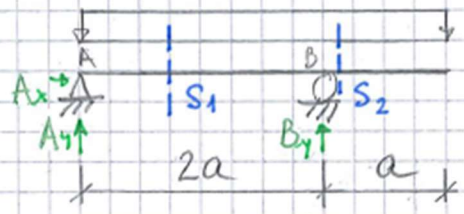
$\Sigma M = 0 : -M(x) - Q.(3a - x) = 0 \Rightarrow M(x) = -Q(3a - x)$

en $x = 2a$: $M = -Qa = -5.2 = -10\,kN.m$

en $x = 3a$: $M = 0$



b).



$q = 10\ kN/m$

$a = 2m$

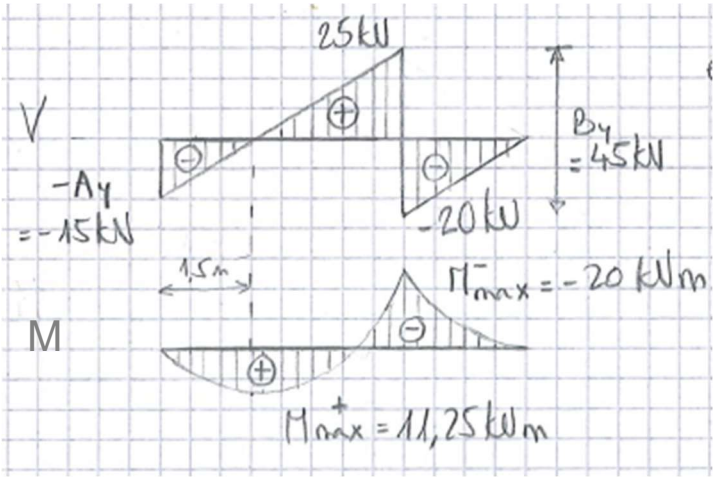
Réactions d'appui

$\Sigma F_x = 0 : A_x = 0$

$\Sigma F_y = 0 : A_y - q(2a+a) + B_y = 0 \Rightarrow A_y = q.3a - B_y$

$\Sigma \mathcal{M}_A = 0 : -q.3a.\frac{3a}{2} + B_y.2a = 0 \Rightarrow B_y = \frac{1}{2}\frac{9}{2}qa = \frac{9}{4}.10.2 = 45kN$

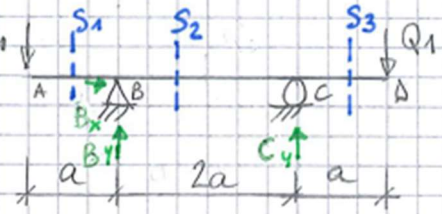
$\Rightarrow A_y = 10.3.2 - 45 = 15kN$



M^+_{max} en $V = 0 \Rightarrow x = \frac{A_y}{q} = \frac{15}{10} = 1,5m$

$M^+_{max} = 15.1,5 - 10.\frac{1,5^2}{2} = 11,25kNm$

c)



$Q_1 = 10\,kN$

$a = 2\,m$

Réactions d'appui:

$\Sigma F_x = 0 : B_x = 0$

$\Sigma F_y = 0 : -Q_1 + B_y + C_y - Q_1 = 0 \Rightarrow B_y = 2Q_1 - C_y$

$\Sigma \Pi_B = 0 : Q_1 . a + C_y . 2a - Q_1 . 3a = 0 \Rightarrow C_y = \frac{1}{2}(3Q_1 - Q_1) = Q_1 = 10\,kN$

$\Rightarrow B_y = 2Q_1 - Q_1 = Q_1 = 10\,kN$